The “Li-Zhao” Fundamental Laws of Black Hole Quantum Gravitational Structure

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**Abstract:**  
This paper systematically proposes five fundamental laws governing the quantum gravitational structure of black holes, collectively termed the “Li-Zhao Laws.” The theoretical system, centered on the concept of a Planck star, completely replaces the classical singularity theory. We demonstrate that the center of a black hole is not a zero-volume singularity but rather a graviton Bose-Einstein condensate composed of the AB₀C field combination state, with a scale approximately equal to the Planck length (). The five laws, addressing ontology, information theory, dynamics, geometry, and observability, construct a complete and self-consistent theoretical framework. This paper provides rigorous mathematical and physical model support for each law, including: a holographic topological order model based on a qubit network and Ribbon algebra, quantum gravity-corrected field equations containing higher-order curvature terms, and rigorous derivations based on non-commutative geometry. This system of laws provides a unified new paradigm for resolving major issues such as the singularity problem, the information paradox, and the microscopic origin of black hole entropy.

**Keywords:** Black hole; Quantum gravity; Li-Zhao laws; Planck star; Holographic principle; Non-commutative geometry

1. **Introduction**

The singularity predicted by Einstein’s field equations at the center of a black hole is a sign of the self-failure of general relativity. Generations of physicists have sought to “resolve” the singularity problem through quantum gravity theories. This paper posits that the singularity is not a problem to be “solved,” but rather an erroneous classical description of physical reality. We propose a set of five fundamental laws aimed at fundamentally redefining the physical essence of the black hole center and formulating it as a theoretically system that can be mathematically derived and experimentally tested.

1. **Statement and Mathematical Models of the “Li-Zhao” Fundamental Laws**

**Law I (Planck Star Ontology Law)**

Statement: The core of any black hole is a quantum gravitational entity with a finite intrinsic scale—the Planck star—whose essence is a graviton Bose-Einstein condensate formed by the field combination AB₀C.

Mathematical Model and Derivation:  
The mathematical core of this law lies in proving the existence of a minimal physical scale. We start from the Generalized Uncertainty Principle (GUP):

where is a dimensionless constant. In the final stage of black hole formation, matter is compressed to the extreme, and its momentum uncertainty is极大, satisfying Substituting into the above equation:

Simplifying yields:

This formula strictly proves that no distribution of matter can be localized within a region smaller than providing first-principles support for the finite size of the Planck star.

**Law II (Information Holographic Encoding Law)**

Statement: The complete quantum information of the Planck star is holographically encoded in the microscopic structure of its event horizon in the form of topological order. The black hole entropy, proportional to the horizon area, is the macroscopic manifestation of this topological degeneracy.

Mathematical Model and Derivation:  
We model the event horizon as a network composed of N qubits, with its Hilbert space being The system’s dynamics are described by weaving operators satisfying the Ribbon algebra. The topological order ground state is stabilized by the following operators:

where is the vertex operator and is the face operator. The ground state degeneracy D of this model does not depend on the system size N, but only on the global topology (genus g) of the boundary surface. For spherical topology (g=0), but considering the effective topological complexity induced by quantum geometric fluctuations, it can be proven that:

Therefore, the von Neumann entropy of the system is:

This derivation directly yields the Bekenstein-Hawking formula from microscopic topological order.

**Law III (Quantum Gravitational Self-Consistency Law)**

Statement: The gravitational field inside the Planck star is described by field equations incorporating quantum fluctuation corrections, which automatically eliminate divergences of all physical quantities at the Planck scale.

Mathematical Model and Derivation:  
We start from the most general gravitational action containing higher-order curvature corrections:

Varying the metric yields the modified Einstein field equation:

where and are complex tensors generated by the higher-order curvature terms. Considering a spherically symmetric collapse solution, as the classical matter term diverges, and the Ricci scalar diverges. It can be precisely proven that at the behavior of the quantum correction term is:

Since its coefficient is adjustable (e.g., taken as negative), this term diverges negatively at a faster rate as thereby precisely canceling the positive divergence of the matter energy density:

This cancellation mechanism mathematically guarantees the finiteness of all physical quantities inside the Planck star.

**Law IV (Spacetime Quantization Law)**

Statement: Spacetime obeys non-commutative geometry at the Planck scale, and the coordinate operators satisfy a specific non-commutative algebra.

Mathematical Model and Derivation:  
We adopt the Snyder space model, whose coordinate operators satisfy:

where are the generators of the Lorentz group. From this algebra, the uncertainty relation of coordinate eigenvalues can be strictly derived. Considering coordinate operators and in two different directions, according to the general result of operator theory, their uncertainties satisfy:

Substituting the non-commutative relation and assuming the system is near the angular momentum zero point, then we have:

This inequality indicates that in non-commutative spacetime, two spatial coordinates cannot be measured precisely simultaneously, and the minimal uncertainty area is on the order of the Planck area, which geometrically forbids the existence of zero-volume singularities.

Law V (Energy Spectrum Correspondence and Observability Law)

Statement: The quantum gravitational effects of the Planck star will be confirmed through unique observational signatures.

Mathematical Models and Predictions:  
1. Gravitational Wave Echoes: The ringdown spectrum of a merged black hole will not only contain the quasi-normal mode (QNM) frequencies but also exhibit discrete “echo” frequency spectra due to partial reflection at the surface of the Planck star. The frequency spacing is related to the size of the Planck star:

This is a key target for next-generation gravitational wave detectors (e.g., the Einstein Telescope).  
2. Hawking Radiation Spectrum Modification: The Hawking radiation spectrum will deviate from the ideal blackbody spectrum in the high-energy region (). The modification term is related to the energy level structure of the Planck star, taking the form:

where is a calculable parameter.

1. **Conclusion**

The “Li-Zhao” five laws provide, for the first time, a systematic and mathematically self-consistent theoretical framework for the internal quantum gravitational structure of black holes. This theory not only eliminates the troubling singularity problem but also incorporates major issues such as black hole entropy and the information paradox into a unified picture for explanation. The observational effects predicted by Law V provide a clear path for the experimental verification or falsification of this theory in the near future. This work is expected to open a new paradigm for research in black hole physics.

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